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THE HEAT-KERNEL AND THE AVERAGE EFFECTIVE POTENTIAL

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Abstract

We discuss the definition of the average effective action in terms of the heat-kernel. As an example we examine a model describing a self-interacting scalar field, both in flat and curved background, and study the renormalization group flow of some of the parameters characterizing its effective potential. Some implications of the running of these parameters for inflationary cosmology are also briefly discussed.

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1. The average effective action Γ_k is the quantum action that describes accurately the behaviour of a given physical system at the momentum scale k. It is obtained via the path integral by integrating in subsequent steps the fields with momenta larger than k. The scale k effectively acts as an infrared cutoff and the flow of Γ_k as k varies is the renormalization group flow.

Originally, these ideas were applied to spin models on the lattice, [1] but they have been subsequently adapted to the continuum, where they have been used to clarify and simplify the notion of renormalizability in quantum field theory. [2, 3] From the computational point of view, the implementation of these ideas presented in [4] is very effective. It has been applied to the study of the phase structures of lower dimensional models as well as to simplified versions of QCD, [5] and to quantum gravity. [6-8]

The purpose of this letter is to elaborate on the definition of the effective action Γ_k using heat-kernel techniques. Within this framework, we shall study the renormalization group flow of some basic parameters characterizing Γ_k in the case of a self-interacting scalar field theory, both in flat and curved spacetime. The analysis of this model is of relevance also for the cosmic evolution of the universe. Indeed, the running of the corresponding average effective potential can have some effects in inflationary cosmology. A brief discussion on some of these effects is reported at the end.

2. Let us consider the theory describing a scalar field $\phi(x)$ interacting via a potential $V(\phi)$. The usual Euclidean effective action Γ is obtained from the functional integral by means of a Legendre transform, using the background field method. At one-loop level, this amounts to the computation of a determinant,

$$\Gamma(\varphi) = \frac{1}{2} \ln \det \mathcal{O}(\varphi) \ , \tag{1}$$

where $\mathcal{O}(\varphi)$ is the elliptic operator that describes the small fluctuations of the quantum field ϕ around the background configuration φ.

The definition of the determinant in (1) requires a regularization, due to the presence of ultraviolet infinities. We assume that some cutoff Λ has been introduced to make (1) meaningful. As we shall see, the choice of regularization will play no role in the considerations that follow.

In general, the average effective action Γ_k can be obtained from Γ by introducing an infrared cutoff at the momentum scale k. The use of a sharp cutoff has some disadvantages; as described in Ref.[4], a smooth cutoff is preferable. To this end, introduce the function

$$P_k(q^2) = \frac{q^2}{1 - e^{-2a(q^2/k^2)^b}} \ , \tag{2}$$

with a and b positive constant parameters. For $|q| \gg k$, the function $P_k(q^2)$ exponentially approaches q^2; on the other hand, it tends to k^2 (when $b = 1$), or diverges (when $b > 1$) for $q^2 \sim 0$.

Consider now the perturbative expansion of Γ in (1) in terms of one-loop graphs; each graph in this expansion involves an integration over a momentum q. The average effective

action Γ_k is then obtained from (1) by making in each momentum integral the substitution:

$$q^\mu \to q^\mu \sqrt{\frac{P_k(q^2)}{q^2}} \ . \tag{3}$$

In this way the propagation of the modes with momenta smaller than the scale k is suppressed. Note that for $b \to \infty$, the function $e^{-2a(q^2/k^2)^b}$ approaches a step function; in this case the modes with $|q| < k$ do not propagate: we are in the case of a sharp infrared cutoff. In conclusion, one can write

$$\Gamma_k(\varphi) = \frac{1}{2} \ln \det{}_k \mathcal{O}(\varphi) \ , \tag{4}$$

where $\det_k$ is the determinant with its one-loop momentum integrations modified as in (3).

A more general definition for Γ_k can be given. It is based on the heat-kernel or proper-time definition of the determinant in (1). [9] To this end, let us introduce the heat equation for the operator $\mathcal{O}$:

$$\frac{d}{dt}K + \mathcal{O} \cdot K = \ 0 \ , \qquad K(x,y)|_{t=0} = \delta(x,y) \ ; \tag{5}$$

it has the formal solution: $K = e^{-t\mathcal{O}}$. Then, one can write:

$$\Gamma = -\frac{1}{2} \int_{1/\Lambda^2}^{\infty} \frac{dt}{t} \, \mathrm{tr} K \ , \tag{6}$$

with Λ the ultraviolet cutoff. As in the perturbative definition of the average effective action given in (4), also in the present case one passes from (6) to Γ_k by introducing an infrared cutoff k. This can be effectively achieved by multiplying the integrand in the rhs of (6) by a universal k-dependent function $F_k(t)$:

$$\Gamma_k = -\frac{1}{2} \int_{1/\Lambda^2}^{\infty} \frac{dt}{t} \, F_k(t) \, \mathrm{tr} K \ . \tag{7}$$

The function F_k has obviously to satisfy some general conditions. First of all, note that dimensional arguments require $F_k(t)$ to depend only on the dimensionless variable $z = k^2 t$: $F_k(t) = F(k^2 t)$. Further, since $\Gamma_k = \Gamma$ at $k = 0$, one must have $F(0) = 1$. Also, $F(z)$ must tend to zero sufficiently rapidly for large z to suppress the small momentum modes. Actually, to enforce this requirement one needs a condition also on the first derivative of $F(z)$: $F'(z) = -z^2 \, f(z)$, with $f(z)$ a positive regular function in the neighborhood of the origin. This last condition is technical, and assures that the renormalization group equation for Γ_k be ultraviolet finite. This is an obvious requirement, since the ultraviolet divergences in the effective action should not be affected by the presence of the cutoff in the infrared region (see also [4]).

3. As a first example of application of the definition (7), let us study the renormalization group flow of the effective potential for a self-interacting scalar field in flat space. Assuming a classical potential $V(\varphi) = \lambda\big(\varphi^2 - \varphi_m^2\big)^2/2$, the small fluctuation operator takes the form:

$$\mathcal{O}(\varphi) = -\partial^2 + 2\lambda(3\varphi^2 - \varphi_m^2) \ . \tag{8}$$

The running potential is also assumed to have the form

$$V_k(\varphi) = \frac{1}{2}\lambda_k\big(\varphi^2 - \varphi_k^2\big)^2 \ ; \tag{9}$$

"irrelevant" terms will be neglected.

We want now to study the evolution in k of the minimum φ_k and of the coupling constant λ_k, which are defined by $V_k'(\varphi_k) = 0$ and $\lambda_k = V_k''(\varphi_k)$ (a prime signifies derivative with respect to φ^2). This is determined by the following coupled differential equations:

$$\begin{aligned} k\frac{\partial\varphi_k^2}{\partial k} &= k^2\,\alpha(k) \ , \qquad & \alpha(k) &= -\frac{1}{k^2\lambda_k}\, k\frac{\partial V_k'}{\partial k}\bigg|_{\varphi_k} \ , \\ k\frac{\partial\lambda_k}{\partial k} &= \beta(k) \ , \qquad & \beta(k) &= k\frac{\partial V_k''}{\partial k}\bigg|_{\varphi_k} \ . \end{aligned} \tag{10}$$

Let us stress that these are approximated evolution equations, consistent with the choice (9). For example a triple derivative V_k''', taking into account the k dependence of the point of definition of λ_k, has been dropped from $\beta(k)$.

Now, from the definition (7) and recalling that for constant background φ, $\Gamma_k(\varphi) = \int d^4x\, V_k(\varphi)$, one obtains

$$\begin{aligned} V_k(\varphi) &= -\frac{1}{2}\int_{1/\Lambda^2}^{\infty}\frac{dt}{t}\,F(k^2t)\int\frac{d^4q}{(2\pi)^4}\,e^{-t\left[q^2+2\lambda(3\varphi^2-\varphi_m^2)\right]} \ , \\ &= -\frac{1}{32\pi^2}\int_{1/\Lambda^2}^{\infty}\frac{dt}{t^3}\,F(k^2t)\,e^{-2t\lambda(3\varphi^2-\varphi_m^2)} \ . \end{aligned} \tag{11}$$

From this expression one can compute the functions $\alpha(k)$ and $\beta(k)$ by taking the appropriate derivatives, and by substituting for the parameters entering the classical potential, λ, φ_m, and for the background φ, their running counterparts λ_k and φ_k. This is the renormalization group improvement, that amounts to recursively upgrade the effective potential at any scale k (for further details, see [4, 5]). Explicitly, one finds:

$$\alpha(k) = \frac{3}{8\pi^2}\,\lambda_k\int_0^{\infty}\frac{dz}{z}\,e^{-4z\lambda_k(\varphi_k^2/k^2)}\,F'(z) \ , \tag{12a}$$

$$\beta(k) = -\frac{9}{4\pi^2}\,\lambda_k^2\int_0^{\infty}dz\,e^{-4z\lambda_k(\varphi_k^2/k^2)}\,F'(z) \ . \tag{12b}$$

Actually, we are interested in approximated expressions for α and β, valid in certain regimes, so that analytic solutions for the system (10) of flow equations can be explicitly

obtained. First, let us assume that $\varphi_k^2/k^2 \ll 1$. In this case, one finds:

$$
\begin{aligned}
k\frac{\partial\varphi_k^2}{\partial k} &= \frac{3A}{8\pi^2}\,k^2\ , \\
k\frac{\partial\lambda_k}{\partial k} &= \frac{9}{4\pi^2}\,\lambda_k^2\ ,
\end{aligned}
\tag{13}
$$

where: $A = \int_0^\infty dz\, z\, f(z)$. Note that the dependence on the cutoff function $F(z)$ is only via the constant A; in particular, the value of β is universal. This is to be expected since in this regime one should recover the results of the standard perturbation theory. Indeed, the equations (13) gives a logarithmic running for quartic self-coupling, while the position of the minimum φ_k scales with k as its dimension, up to logarithmic corrections.

In the region for which $\varphi_k^2/k^2 \gg 1$, one obtains:

$$
\begin{aligned}
k\frac{\partial\varphi_k^2}{\partial k} &= \frac{3B}{128\pi^2}\,\frac{k^2}{\lambda_k^2}\,\left(\frac{k^2}{\varphi_k^2}\right)^2\ , \\
k\frac{\partial\lambda_k}{\partial k} &= \frac{9B}{128\pi^2}\,\frac{1}{\lambda_k}\,\left(\frac{k^2}{\varphi_k^2}\right)^3\ ,
\end{aligned}
\tag{14}
$$

with $B = f(z)|_{z=0}$. In this regime, φ_k and λ_k tend to constants as k goes to zero. This behaviour and the one dictated by (13) coincide with the ones computed in Refs.[4, 8], using standard methods.

The renormalization flow equations (13) and (14) depend on the choice of the infrared cutoff function F through the constants A and B. A particularly interesting explicit choice for F can be obtained using the definition (2), (3), (4) for the average effective action. In this case one obtains:

$$
F(z) = \int_0^\infty dx\, x\, \exp\left[-\frac{x}{1-e^{-2a(x/z)^b}}\right]\ . \tag{15}
$$

This expression essentially coincides with the trace of the heat kernel for the operator $P_k(-\partial^2)$. One can check that this function satisfies all the properties we have previously required for F. With this particular choice, one easily gets:

$$
A = \frac{1}{(2a)^{1/b}}\,\Gamma(1+1/b)\ , \qquad B = \frac{1}{(2a)^{3/b}}\,\Gamma(1+3/b)\,\zeta(3/b)\ . \tag{16}
$$

4. The definition (7) for the average effective action has the advantage that it can be equally well applied to field theory models on curved spacetime backgrounds. As an example, let us consider the theory for a scalar field φ, with Euclidean action:

$$
S(\varphi) = \int d^4x\sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi + \frac{1}{2}\xi\,R\,\varphi^2 + V(\varphi)\right)\ , \tag{17}
$$

propagating on a background with metric $g_{\mu\nu}$ and scalar curvature R. We take again a potential V of the form (9) and concentrate on the study of the renormalization group flow of the corresponding parameters λ_k, ξ_k and of the minimum φ_k.

The explicit expression for Γ_k can be obtained applying Eq.(7), where now K is the heat kernel for the operator

$$\mathcal{O}(\varphi) = \widetilde{\mathcal{O}} + 2\lambda(3\varphi^2 - \varphi_m^2) \ , \qquad \widetilde{\mathcal{O}} = -\nabla^2 + \xi\, R \ . \tag{18}$$

More explicitly, calling $\widetilde{K}$ the heat kernel for $\widetilde{\mathcal{O}}$, one has

$$\Gamma_k(\varphi) = -\frac{1}{2}\int_{1/\Lambda^2}^{\infty} \frac{dt}{t}\, F(k^2 t)\, e^{-2t\lambda(3\varphi^2-\varphi_m^2)}\, \mathrm{tr}\widetilde{K} \ . \tag{19}$$

In this case, the trace of $\widetilde{K}$ can not be computed in closed form. However, it has a Laurent series expansion around $t = 0$:

$$\mathrm{tr}\widetilde{K} = \sum_{n=0}^{\infty} t^{n-3} \int d^4x\, \sqrt{g}\, \tilde{b}_n \ , \tag{20}$$

where $\tilde{b}_n$ are the so called heat kernel coefficients. [9] They can be explicitly computed in terms of powers of the curvature tensors and their derivatives, *e.g.* $\tilde{b}_0 = 1/16\pi^2$, $\tilde{b}_1 = (1/16\pi^2)(1/6 - \xi)\, R$.

Notice that in writing (19), we have implicitly taken F, the function that introduces the infrared scale k, to be independent of the background metric $g_{\mu\nu}$. Although other approaches are certainly conceivable, this choice allows an easy evaluation of the various beta-functions.

We shall concentrate on the study of the effective potential $V_k(\varphi)$ which is obtained by taking a constant background configuration φ. Also, for the sake of simplicity, we shall take the curvature R to be constant. In this situation, the parameters φ_k and λ_k are defined as before, while we introduce the non-minimal coupling ξ_k as the derivative with respect to R of $V_k'(\varphi_k)$, at $R = 0$. Then, the momentum flows of φ_k and λ_k are given again by the equations (10), while for ξ_k one finds:

$$k\frac{\partial \xi_k}{\partial k} = \gamma(k) \ , \qquad \gamma(k) = k\frac{\partial}{\partial k}\frac{\partial V_k'}{\partial R}\bigg|_{\varphi_k,\ R=0} \ . \tag{21}$$

The expressions for the quantities $\alpha(k)$, $\beta(k)$ and $\gamma(k)$ can now be explicitly computed as a series expansion in R. One finds:

$$\alpha(k) = 6\lambda_k \sum_{n=0}^{\infty} \frac{b_n(\xi_k)}{(k^2)^n} \int_0^{\infty} dz\, z^{n-1}\, e^{-4z\lambda_k(\varphi_k^2/k^2)}\, F'(z) \ , \tag{22a}$$

$$\beta(k) = -36\lambda_k^2 \sum_{n=0}^{\infty} \frac{b_n(\xi_k)}{(k^2)^n} \int_0^{\infty} dz\, z^{n}\, e^{-4z\lambda_k(\varphi_k^2/k^2)}\, F'(z) \ , \tag{22b}$$

$$\gamma(k) = \frac{3\lambda_k}{4\pi^2}\left(\frac{1}{6} - \xi_k\right) \int_0^{\infty} dz\, e^{-4z\lambda_k(\varphi_k^2/k^2)}\, F'(z) \ . \tag{22c}$$

As in the case of flat space, the flow equations (10) and (21) simplify if one takes the limit for which φ_k^2/k^2 is very small or very large, and further assume that the curvature R to be small with respect to φ_m and k. Let us first consider the case $\varphi_k^2/k^2 \ll 1$. Keeping only the dominant terms and defining the convenient combination $\eta_k = 1/6 - \xi_k$, one can write:

$$k\frac{\partial\varphi_k^2}{\partial k} = \frac{3}{8\pi^2}\,k^2\left[A + \eta_k\frac{R}{k^2}\right]\ , \tag{23a}$$

$$k\frac{\partial\lambda_k}{\partial k} = \frac{9}{4\pi^2}\,\lambda_k^2\left[1 + C\eta_k\,\frac{R}{k^2}\right]\ , \tag{23b}$$

$$k\frac{\partial\eta_k}{\partial k} = \frac{3}{8\pi^2}\,\lambda_k\eta_k\ , \tag{23c}$$

where $C = -\int_0^\infty dz\,F(z)$. Notice that for $R = 0$ the equations (23a) and (23b) reduce to those in (13). Further, the value of γ, *i.e.* the rhs of (23c), reproduces the standard perturbative result, [10] as expected. The equations (23) give the dominant logarithmic running for the coupling constants λ_k and ξ_k, plus subleading logarithmic corrections depending on the curvature. Similarly, φ_k^2 scales as k^2, with additional logarithmic corrections proportional to R.

In the complementary region, $\varphi_k^2/k^2 \gg 1$, one obtains instead

$$k\frac{\partial\varphi_k^2}{\partial k} = \frac{3B}{128\pi^2}\,\frac{k^2}{\lambda_k^2}\,\left(\frac{k^2}{\varphi_k^2}\right)^2\,\left[1 + \frac{\eta_k}{2\lambda_k}\,\frac{R}{\varphi_k^2}\right]\ , \tag{24a}$$

$$k\frac{\partial\lambda_k}{\partial k} = \frac{9B}{128\pi^2}\,\frac{1}{\lambda_k}\,\left(\frac{k^2}{\varphi_k^2}\right)^3\,\left[1 + \frac{3\eta_k}{4\lambda_k}\,\frac{R}{\varphi_k^2}\right]\ , \tag{24b}$$

$$k\frac{\partial\eta_k}{\partial k} = \frac{3B}{256\pi^2}\,\frac{1}{\lambda_k^2}\,\left(\frac{k^2}{\varphi_k^2}\right)^3\,\eta_k\ . \tag{24c}$$

The running of φ_k^2, λ_k and ξ_k is suppressed by powers of k^2/φ_k^2, and actually stops for $k \sim 0$. For small k, one can approximate the couplings λ_k and ξ_k with their values at $k = 0$. In this case, one further has:

$$\varphi_k^2 = \varphi_0^2\left\{1 + \frac{B}{256\pi^2}\,\frac{1}{\lambda_0^2}\,\left(\frac{k^2}{\varphi_0^2}\right)^3\,\left[1 + \frac{\eta_0\lambda_0}{2}\left(\frac{R}{\varphi_0^2}\right)\right]\right\}\ . \tag{25}$$

5. The previous results on the running of the minimum φ_k of the effective potential could have some consequences on the cosmic evolution of the universe. In standard inflationary cosmology, the field φ is a Higgs field of a Grand Unification Model, that undergoes a spontaneous symmetry breaking. It is responsible of inflation via its slow rolling from the configuration $\varphi = 0$ to the true minimum of the effective potential.

Usually, one treats this evolution semiclassically, by adding the Einstein term to the action (17), and assuming for the metric $g_{\mu\nu}$ a Friedman form. The approximated cosmic-time evolution of the scale factor $a(t)$ of the universe is then given by ($H \equiv \dot{a}/a$): [11]

$$H^2 = \frac{8\pi G_N}{3}\,V(0)\ , \tag{26}$$

where G_N is Newton's constant and $V(0)$ is the value of the effective potential for the matter field φ at the starting point $\varphi = 0$. We use the convention that gives to a the dimensions of length.

In the standard approach, $V(0)$ is considered constant, so that from (26) one has the inflationary exponential behaviour:

$$a(t) \sim a(0)\, e^{H\, t} \ . \tag{27}$$

However, things change if we take into account the running of the effective potential. In this case, from (9) one has $V_k(0) = \lambda_k \varphi_k^4/2$. As previously explained, k must coincide with the characteristic scale of the phenomena under study. We are following the evolution of the scale factor a of the universe; it is therefore natural to identify: $k \sim 1/a$. Further, when inflation starts, k is of the order of Planck's mass M_P, and therefore very large with respect to the masses of all ordinary particles. Indeed, we are studying the cosmic evolution immediately after Planck's time, *i.e.* in the regime where semiclassical considerations are justified. As shown previously, for such large values of k, φ_k scales approximately as k, with a proportionality constant c of order one, while λ_k runs logarithmically. Neglecting for simplicity this mild change of λ_k, Eq.(26) becomes:

$$H^2 = \frac{4\pi\lambda G_N}{3}\, \frac{c^4}{a^4} \ . \tag{28}$$

In this case, one no longer has an inflationary behaviour, since: $a(t) \sim t^{1/2}$.

This conclusion, based on the simplified equation (26), is a little crude, but clearly indicates a change in the evolution of the cosmic factor $a(t)$ due to the running of the effective potential for φ. More accurate conclusions can be obtained by considering together the equations of motion of both the field φ and the scale factor a. As a starting action we take again (17) plus the Einstein term, but for simplicity we limit our considerations to the case $\xi = 0$.

Assuming an inflationary regime, for which $|\dot{H}| \ll H^2$, and a slow changing homogeneous field φ, one has: [11]

$$3H\dot{\varphi} + 2\lambda\varphi\big(\varphi^2 - \varphi_k^2\big) = 0 \ , \tag{29a}$$

$$H^2 = \frac{4\pi\lambda G_N}{3}\, \big(\varphi^2 - \varphi_k^2\big)^2 \ . \tag{29b}$$

Here again we neglect the running of the coupling constant λ. Irrespectively of the value of φ_k, the evolution of φ is exponentially dumped:

$$\varphi(t) = \varphi(0)\, e^{-h\, t} \ , \qquad h = \sqrt{\frac{\lambda}{3\pi\, G_N}} \ . \tag{30}$$

Then, using again $\varphi_k = c/a$, (29b) gives the following behaviour for $a(t)$:

$$\begin{aligned} a^2(t) =& a^2(0)\, e^{2\pi G_N\left(\varphi^2(0)-\varphi^2(t)\right)} \\ &\times \left\{1 - \frac{2\pi c^2 G_N}{a^2(0)} e^{-2\pi G_N \varphi^2(0)}\ \Big[E\Big(2\pi G_N\varphi^2(0)\Big) - E\Big(2\pi G_N \varphi^2(t)\Big)\Big]\right\} \ , \end{aligned} \tag{31}$$

where $E(x)$ is the exponential-integral: $E(x) = \int^x dz\, e^z/z$.

At the starting point, immediately after Planck's time, one can reasonably assume: $V_k(\varphi(0)) \sim M_P^4$. For λ small, this implies that $\varphi(t)$ starts at a value $\varphi(0) \sim \lambda^{-1/2} M_P \gg M_P$, and varies very little over the interval: $\Delta t \sim (\sqrt{\lambda}\, M_P)^{-1}$. In this interval, the scale function $a(t)$ has an inflationary exponential behaviour due to the first factor in (31), which is however stopped immediately after by the second factor.

In this framework there is no need for an additional mechanism to stop inflation: everything is already encoded in the approximated equations (31), provided the renormalization flow of φ_k is taken into account. Further detailed analysis is certainly needed to confirm this behaviour. Nevertheless, we find these preliminary results worth of consideration.

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